ALL SAINTS COLLEGE LIBRARY TRIVANDRUM

(Pages : 4) H – 6259

Reg. No. : .....

Name : .....

Fifth Semester B.Sc. Degree Examination, December 2019

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1541 : REAL ANALYSIS — I

(2014 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry 1 mark each.

1. Define an ordered set.
2. Find the supremum of $E = \{1, 1/2, 1/3, ...\}$
3. State the Archimedean property of R.
4. Define a bounded set of real numbers.
5. Give an example of an infinite set which has no limit points.
6. Define a Cauchy sequence.
7. Show by an example that every bounded sequence need not be convergent.
8. State the necessary condition for convergence of a series.

P.T.O.

9. Find $\lim_{x \to 0} \frac{x}{|x|}$.

10. Show that the sequence $\{(-1)^n\}$ is not convergent.

SECTION – II

Answer **any eight** questions (11-22). Each question carries **2** marks.

11. Show that there exists no rational number whose square is 3.

12. State the order properties of R.

13. State the triangle inequality.

14. Show that between any two real numbers there is a rational one.

15. Show that a convergent sequence has only one limit.

16. Show that the sequence $\{(-1)^n\}$ is not convergent.

17. Check whether the sequence $\{2n + (-1)^n\}$ monotonic or not.

18. Show that a Cauchy sequence of real numbers is bounded.

19. Show that $\lim_{n \to \infty} \frac{\sin n}{n} = 0$.

20. Find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

21. Show that $\sum_{1}^{\infty} \frac{1}{\sqrt{n+1}}$ is divergent.

22. Show that $\sum_{1}^{\infty} \frac{1}{n(n+1)} = 1$.

SECTION – III

Answer **any six** questions (23- 31). Each question carries **4** marks.

23. State and prove density theorem.

24. State squeezing theorem for sequences.

25. Give an example of sequences $\{x_n\}$ and $\{y_n\}$ which are not convergent but the sequence $\{x_n y_n\}$ is convergent.

26. Show that $\sum_{1}^{\infty} \frac{\cos n}{n^2}$ is convergent.

27. Show that the sequence $\left(\frac{2n+3}{3n-2}\right)$ is convergent.

28. Test for convergence the series $\sum_{1}^{\infty} n^{\frac{1}{n}}$.

29. Show that $\lim_{n\to\infty} n^{1/n} = 1$.

30. Show that $\lim_{x\to 0} x \sin(1/x) = 0$.

31. Discuss the convergence of the series whose n^{th} term is given by

$u_n = (10n+1)/(n(n+1)(n+2))$.

SECTION – IV

Answer **any two** questions (32-35). Each question carries **15** marks.

32. (a) If $\{I_n\}$ is a sequence of intervals in R such that $I_n \supseteq I_{n+1} (n = 1, 2, 3, \ldots)$ then $\bigcap_{n=1}^{\infty} I_n$ is non empty.

(b) Prove that a monotonic sequence is convergent if and only if it is bounded.

33. (a) Show that $\sum_{1}^{\infty} \frac{1}{n^p}$ converges if p > 1 and diverges if $p \leq 1$.

(b) State the sequential criterion for a limit of a function with giving an example.

34. (a) Let $\{x_n\}$ and $\{y_n\}$ be two sequences of real numbers such that $\lim x_n = x$ and $\lim y_n = y$. Show that

$$\lim(x_n y_n) = \lim x_n \cdot \lim y_n = xy .$$

(b) Prove that every bounded infinite set has at least one limit point.

35. (a) Establish Cauchy criteria for convergence of sequence of real numbers.

(b) Define a contractive sequence and show that every contractive sequence is a convergent sequence.
